EFFICIENT ALGORITHM FOR STEADY-STATE STABILITY ANALYSIS OF LARGE ANALOG/RF CIRCUITS

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Abstract -- **This paper presents a method for the investigation of AC and large signal steady-state stability of electrical (analog/RF) circuits. In both cases stability/instability are detected through fast calculation and analysis of circuit poles. Thanks to iterative algorithms (Krylov-Subspace methods) applied to Modified Nodal formulation of conversion matrices a selection of poles is computed, allowing the method to be applied to large size circuits of any kind of topology.**

I. Introduction

Simulation tools developed on the basis of Harmonic Balance (HB) and associated techniques allow today the steady state analysis with reasonable computing time and memory requirement for large size nonlinear RF circuits. These techniques allow to reliably simulate large size forced/autonomous circuits under single tone excitation or arbitrary modulated carrier excitation in presence of noise sources.

Nevertheless this panoply of tools intended for commercial CAD applications does not allow to predict systematically instabilities that can occur, in particular for large size nonlinear circuits. From this viewpoint, the lack of efficient algorithms to detect possible instabilities makes any real design not very reliable.

Theoretical studies and efficient methods to investigate linear instability behavior have been established a long time ago [1-3] and more recently algorithms for nonlinear instabilities of small size circuits have been proposed [4-8]. Unfortunately, expanding these suggested methods to large size circuits and general topology becomes quickly inextricable and inefficient due to large CPU times and storage requirements. These factors are often incompatible with CAD requirements.

In this paper, a novel, efficient and rigorous method is proposed to analyze circuit instabilities, valid for both linear and nonlinear circuits. It can handle large size circuits of any topology, using the Modified Nodal formulation, conversion matrices and Krylov-subspaces for the system resolution.

Section II of this paper describes the theory of poles in linear & nonlinear electrical networks, section III discusses the mathematical algorithms to compute the poles and their implementations. The method is illustrated with three different examples.

II. Poles in electrical networks

A. The AC case:

Let us consider an electrical circuit in DC equilibrium described by Modified Nodal equations. Now let us stimulate the circuit with a small signal u_p of frequency f_p around its DC operating point. The circuit response x_p is the solution of the linear system:

$$Y_p x_p = u_p \tag{1}$$

Where Y_p is the admittance matrix of the circuit:

$$Y_p = G_{dc} + j2\pi f_p C_{dc} \tag{2}$$

Using the circuit Modified Nodal equations and the classical associated equations, G_{dc} and C_{dc} are respectively the equivalent conductance and capacitance matrices, representing the static and dynamic parts of the circuit.

If we consider the system output $a^T.x$ for a single input u_k, the network transfer function $H(s)$ is [9]:

$$H(s) = \frac{a^T x}{u_k} = \frac{\det(\tilde{Y}_s(k,a))}{\det(Y_s)} \tag{3}$$

$$\tilde{Y}_s(k,a) = Y_s + e_k(a^T - e_k{}^T Y_s) \tag{4}$$

e_k and a are incident vectors selecting respectively the input and the desired output.

The network poles p_i are the values of s ($s=\sigma + j2\pi f$) such that $\det(Y_s) = 0$. They are the solution of the generalized eigenvalue problem:

$$(G_{dc} + p_i C_{dc})X = 0 \tag{5}$$

that can be formulated as:

$$G_{dc}X = -p_i C_{dc} X \tag{6}$$

B. The large signal Steady-State case:

Now suppose that our circuit is in large signal periodic steady-state conditions (it is driven by large signal periodic stimuli S_0 of fundamental frequency f_0 and all the circuit nodes $\boldsymbol{V}$ are in steady-state). The circuit can be described by the following Modified Nodal equations in the frequency domain:

$$F(V) = I_k(V) + j2\pi k f_0 Q_k(V) + S_0 \quad (7)$$

I_k and Q_k respresent the k^{th} Fourier components of the currents and charges at steady-state respectively. ($-N \leq k \leq N$) where N is the number of harmonics used to compute the steady-state.

Let us stimulate the circuit with a small signal u_p of frequency f_p around its large signal steady-state. The circuit response X_p is thus [10]:

$$J_p X_p = u_p \quad (8)$$

J_p is known as the "frequency conversion matrix":

$$J_p = G_{ss} + j2\pi f_0 K C_{ss} + j2\pi f_p C_{ss} \quad (9)$$

It is calculated from the Harmonic Balance (HB) Jacobian matrix:

$$J_0 = G_{ss} + j2\pi f_0 K C_{ss} \quad (10)$$

K is a diagonal matrix containing the harmonic indices. G_{ss} and C_{ss} are block circulant matrices built from the Fourier components of the Modified Nodal conductance and capacitance matrices. All details concerning these matrices can be found in [10].

Similar to the AC case, the steady-state network poles p_i are the value of s ($s = \sigma + j2\pi(kf_0+f)$) such that $\det(J_s) = 0$. They are the solution of the generalized eigenvalue problem:

$$J_0 X = - p_i C_{ss} X \quad (11)$$

C. Poles and circuit stability:

A circuit is stable if all the poles have negative real parts (algebraic first Lyapunov criterion). In other words a circuit is unstable if at least one of its poles has a positive real part. Therefore a stability analysis can be based on the identification of poles having positive real parts.

Regarding the Pole/Zero cancellation problem [11], pole computation is in principle more desirable than Bode magnitude and phase plots that are typically used by designers. For instance, if there is a closely spaced pole-zero pair, a Bode plot will not reveal it whereas pole computation will do.

D. Comments:

In the AC case, the circuit response x_p, solution of equation (**1**) is at the same frequency f_p as the input signal u_p. In the large signal steady-state case, due to non-linearities, the circuit will exhibit frequency conversion. It means that the circuit response X_p, solution of equation (**8**) contains components at the different frequencies: $f = k\ f_0 + f_p$ ($-N \leq k \leq N$) where N is the number of harmonics used to compute the steady-state. This implies that each pole $p = p\_r + jp\_i$ will appear in conjunction with a family of poles of the form: $p\_r + j(k\ f_0 + p\_i)$. This phenomenon is illustrated in the first example below.

The equations (**6**) and (**11**) are formulated thanks to the Modified Nodal formulation of the circuit. If the circuit contains frequency dependent elements such as transmission lines or S parameter blocks their characteristics are approximated either by a s domain polynomial or rational function, to be handled by the method.

III. Numerical methods for pole computation

We have seen in the previous paragraph that one needs to solve the following generalized eigenvalue system to find the circuit poles:

$$AX = \lambda_i BX \quad (12)$$

In the AC case $A = G_{dc}$ and $B = -C_{dc}$ and in the large signal steady-state case $A = J_0$ and $B = -C_{ss}$.

QZ is the "standard" method to solve system (**12**) [9][12]. It is a numerically stable algorithm and its computational cost is $O(n^3)$ where n is the size of the system. This method computes all the eigenvalues (poles) λ_i and is limited to moderate size systems (n < few hundred). It could be used for AC stability analysis but it is clearly untractable for large signal steady-state stability of RF circuits (which system size n may reach millions).

It should be noticed that it is not necessary to compute all the circuit poles, because only the right most is/are required to detect instability. The Krylov subspace iterative methods are good candidates to solve partial eigenvalue problems [13]. They are able to find selected eigenvalues with a computational cost close to $O(n)$. Basic Arnoldi or Lanczos algorithms allow to find eigenvalues with the largest (or smallest) module. They are less suitable to our problem than a variant of Arnoldi method (that uses Chebyshev acceleration), directed to determine the right-most eigenvalues [14-15]. The Arnoldi method with Chebyshev acceleration is reasonably easy to code, and mathematical libraries containing such routines exist [16-17]. It means that the implementation of stability analysis inside a general purpose Spice-like or HB-based simulator is relatively simple.

A. The AC case:

The Arnoldi/Chebyshev algorithm solves "standard" eigenvalue systems:

$$AX = \lambda X \quad (13)$$

The system (**6**) must then be transformed ($A=-C_{dc}^{-1}G_{dc}$ or $A=-G_{dc}^{-1}C_{dc}$) to be solved. As C_{dc} is often singular, the second transformation ($A=-G_{dc}^{-1}C_{dc}$) is preferred, corresponding to the inverted problem ($\lambda_i = 1/\ p_i$). The Arnoldi/Chebyshev algorithm does not require explicitely the A matrix, but only matrix-vector products.

B. The large signal steady-state case:

As for the AC case the generalized eigenvalue system (**11**) has to be transform into a "standard" eigenvalue system of form (**13**), with:

$$\boldsymbol{A}=-\boldsymbol{C}_{ss}^{-1}\boldsymbol{J}_0 \quad \textbf{(14)}$$

Or:

$$\boldsymbol{A}=-\boldsymbol{J}_0^{-1}\boldsymbol{C}_{ss} \quad \textbf{(15)}$$

$\boldsymbol{J}_0$ and $\boldsymbol{C}_{ss}$ are matrices function of the steady-state operating point. They have a special structure (Block Toeplitz/Circulant) and products with vectors can be performed efficiently with a computational cost close to $O(\boldsymbol{n}.\log(\boldsymbol{n}))$ [18].

AC and large signal steady-state stability analyses have been implemented in an advanced version of the electrical (analog/RF) simulator Eldo RF using QZ and Arnoldi/Chebyshev algorithms. Eldo RF computes the steady-state with a HB based algorithm.

The DC stability analysis is used to check the stability of forced circuits and also to provide an estimation of the oscillation frequency for autonomous circuits (oscillators). The large signal steady-state stability analysis is used to check the stability of forced and autonomous circuit steady-state results. The implemented analysis has been intensively tested with different family of circuits and revealed that most of the circuits showing steady-state convergence difficulties contained instabilities.

IV. Examples

The first example is a LNA circuit operating in the 100MHz range. The presented method is used to check AC stability of the circuit. We find a pair of complex conjugate poles (–4.9487E+05 +/- j 6.2830E+09) corresponding to the resonator. The large signal steady-state stability has been performed on this circuit to compute all the circuit poles.

The steady-state is stable, all the poles are located in the left half part of the plan and we actually find a collection of poles of the form $(-4.9487E+05 + j(2.\pi.k.f_0 \pm 6.2830E+09)$.

The second example is an oscillator. The steady-state stability analysis provides a collection of poles having the same positive real part and imaginary parts close to $2.\pi.k.f_0$. We first thought that these poles correspond to the inherent instability related to the oscillation frequency. In fact after investigation of the AC stability we realized that the circuit contained 2 oscillation frequencies (2 pairs of complex conjugate poles with positive real parts, the first corresponding to the required oscillation frequency and the second being a spurious one). After redesigning the circuit to remove the undesired instability, we finally realized from the large signal steady-state stability results that no more poles appear in the right half part of the plan.

The third example is a receiver circuit driven by multitone signals and containing a RF amplifier followed by an I/Q mixer (see figure 1). The amplifier bias is controlled by the power at the output of the mixer. The complete circuit netlist contains about 1000 components (linear & nonlinear devices) and 600 nodes. A typical analysis of this circuit consists of a 2 tone (1 RF & 1 LO) steady-state analysis with a sweep of the RF power (from –35dBm to 0dBm) in order to characterize the 1dB compression point. For RF power level above –10dBm the steady-state algorithms has difficulties to converge. We thus compute the stability for each large signal steady-state results and find out that the circuit becomes unstable at P_{RF}=-10dBm (i.e. the real part of a collection of poles moves from the left to the right half part of the plan and crosses 0 at –10dBm).

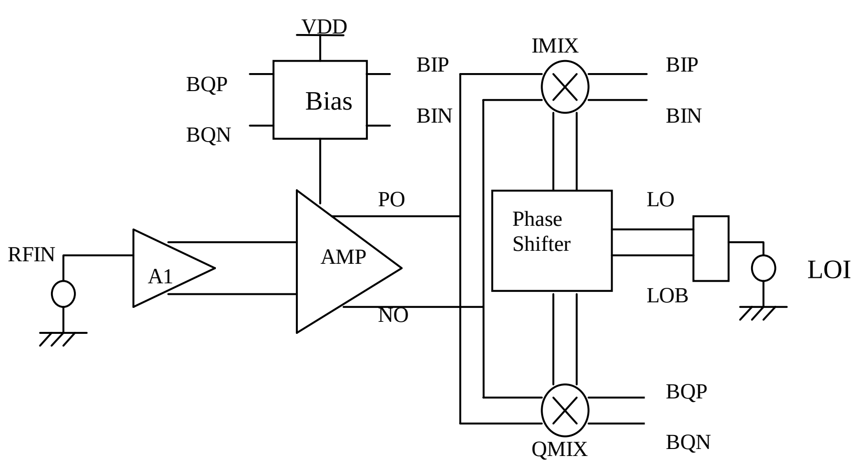

Figure 1: Block diagram of the receiver circuit.

V. Conclusion

We present a new method to compute the linear (AC) and nonlinear (large signal multitone steady-state) stability of electrical (analog/RF) circuits. This method is based on a Modified Nodal formulation of the circuit equations, frequency domain conversion matrices and the computation of selected poles using Krylov-subspace iterations. The method can handle large circuits of any kind of topology and is easily implementable in a general purpose/HB based circuit simulator. It has been introduced in a development version of the electrical simulator Eldo RF and is illustrated in this paper by various examples. The described method represents (from the author point of view) the state-of-the-art in term of large signal steady-state analysis allowing its use in CAD tools to verify large size real designs.

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